

Mini-workshop on Higgs bundles

Date: 27th-28th May 2024. (2024年5月27-28日)

Place: Lecture Room E404 in Graduate School of Science Building E in Osaka University (Toyonaka Campus). (大阪大学理学部 E404 講義室 (豊中キャンパス))

Program

27th May (Monday)

10:00–11:00 Laura Schaposnik (University of Illinois)

An introduction to Higgs bundles and their integrable system I

11:30–12:30 Laura Schaposnik (University of Illinois)

An introduction to Higgs bundles and their integrable system II

14:30–15:30 Natsuo Miyatake (Mathematical Science Center for Co-creative Society, Tohoku University)

Harmonic metrics on cyclic Higgs bundles, subharmonic functions, and entropy

16:00–17:00 Mengxue Yang (Kavli IPMU, The University of Tokyo)

Conformal limit on Cayley components

28th May (Tuesday)

10:00–11:00 Laura Schaposnik (University of Illinois)

An introduction to Higgs bundles and their integrable system III

Information

This workshop will be held as a pre-seminar for our conference "New developments in Kobayashi-Hitchin correspondence and Higgs bundles" from 5th-9th August 2024 in Osaka Metropolitan University.

Organizers

- Yoshinori Hashimoto (Osaka Metropolitan University)
- Masataka Iwai (Osaka University)
- Hisashi Kasuya (Osaka University)
- Natsuo Miyatake (Mathematical Science Center for Co-creative Society, Tohoku University,)

Supports

- JSPS KAKENHI 19H01787 Grant-in-Aid for Scientific Research (B)
- JSPS KAKENHI 24K16912 Grant-in-Aid for Early Career Scientists.

Abstract

Laura Schaposnik (University of Illinois)

An introduction to Higgs bundles and their integrable system.

During the mini-course we will introduce Higgs bundles and their integrable system by first considering the basic definitions, and slowly introducing the Hitchin fibration. We will then look at the Hitchin fibration for different groups and see how dualities arise (be them from mirror symmetry, or from other correspondences such as low rank isogenies). Finally, we will dedicate the last talk to the introduction of some particular Lagrangians in the moduli space of Higgs bundles giving branes, whose understanding can lead to insights in representation theory (about representations of 3-manifolds, equivariant representations, etc).

Natsuo Miyatake (Mathematical Science Center for Co-creative Society, Tohoku University)

Harmonic metrics on cyclic Higgs bundles, subharmonic functions, and entropy

Let X be a Riemann surface and $K_X \rightarrow X$ the canonical bundle. For each integer $r \geq 2$, each $q \in H^0(K_X^r)$, and each choice of the square root $K_X^{1/2}$ of the canonical bundle, we canonically obtain a Higgs bundle, which is called a cyclic Higgs bundle. In this talk, I will introduce several new notions regarding cyclic Higgs bundles. First, I will introduce the notion of cyclic Higgs bundles with multi-valued Higgs fields and their associated Hitchin equation. Second, I will introduce a generalization of the Hitchin equation for cyclic Higgs bundles associated with a quasi-subharmonic function, obtained by infinitely increasing the degree of multivalence of the multi-valued Higgs field. Third, I will further generalize this generalized Hitchin equation to complex higher-dimensional manifolds. Finally, I will propose a new concept, which I call cyclic entropy, defined using the solution to the generalized Hitchin equation for cyclic Higgs bundles associated with a quasi-subharmonic function. One of the motivations for introducing these new notions comes from weighted potential theory, including the theory of the asymptotic behavior of sections of holomorphic line bundles. I will present the results obtained so far regarding these new concepts and discuss their potential for further development.

Mengxue Yang (Kavli IPMU, The University of Tokyo)

Conformal limit on Cayley components

In 2014, Gaiotto conjectured that there is a biholomorphism between Hitchin components and spaces of opers on a punctured sphere via a scaling limit called the \hbar -conformal limit. On a compact Riemann surface of $g \geq 2$, this biholomorphism has been proven in 2016. Motivated by the study of higher Teichmüller spaces, we may view the Hitchin components as a part of a larger family of special components called Cayley components. I will talk about the Cayley components and propose their conformal limit to be the generalized notion of opers of Collier—Sanders.