

OpenAI Math Digest

CATALOGUE 052

Tangent-bundle splittings and universal covers

Catalogue synthesis & 2 proof overviews

Independent reading guide · English edition

Edition: 2026-10-13

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AI-generated explanations may contain errors. Check important claims and proofs against the original manuscripts. This booklet collects the website's explanations, not the original papers. Expert review and formal verification have not been performed.

Contents

The synthesis comes first, followed by all 2 overviews in official catalogue order. The contents and citations are clickable. Wide tables are arranged as entries for print; the claims and source references are retained.

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Original-manuscript snapshot: 1581d4cbdd69a0faf34b2089614fe2a814316a6a

Tangent-bundle splittings and universal covers

These two papers separate obtaining integrability for a specified holomorphic splitting $T_X = E_1 \oplus E_2$ from passing to a global product. In the compact Kähler setting, integrability is assumed and the ordinary universal cover is split. In the rationally connected projective setting, integrability is proved before splitting X itself.

1. Papers in official catalogue order

These summaries give the principal scope; consult the cited original results for the complete hypotheses and statements.

Catalogue 052 · 2026-10-13 checked source snapshot

#

01

Manuscript and overview

Universal-cover splitting for compact Kähler manifolds →

Short name: Kähler universal-cover splitting

September 23, 2026 · 33 pages

Original paper (GitHub) ↗

Principal claim

Two specified integrable tangent summands on a compact Kähler manifold yield a compatible product of its ordinary universal cover. Applications cover non-uniruled projective manifolds and nef and big canonical bundles.

Theorem 1.1 · p. 3; Corollaries 1.2–1.3 · pp. 3–4 ↗

Role and available overview

Globalizes local products through holomorphic-box extension, grids retaining full-polydisk axes, and continuation of inverse germs.

Read the proof overview →

#

02

Manuscript and overview

Integrability of split tangent bundles on rationally connected manifolds →

Short name: Integrability on rationally connected manifolds

September 23, 2026 · 16 pages

[Original paper \(GitHub\)](#) ↗

Principal claim

A specified tangent two-splitting on a smooth rationally connected projective manifold is integrable. Höring gives a compatible product of X itself; the result also supplies structure along general rationally connected fibres.

[Theorem 1.1 · p. 1](#); [Corollary 1.2 · p. 2](#); [Corollary 5.1 · pp. 12–13](#) ↗

Role and available overview

Extends rational curves to tangent boxes; projected-bracket transport and two-point evaluation annihilate the Frobenius obstruction.

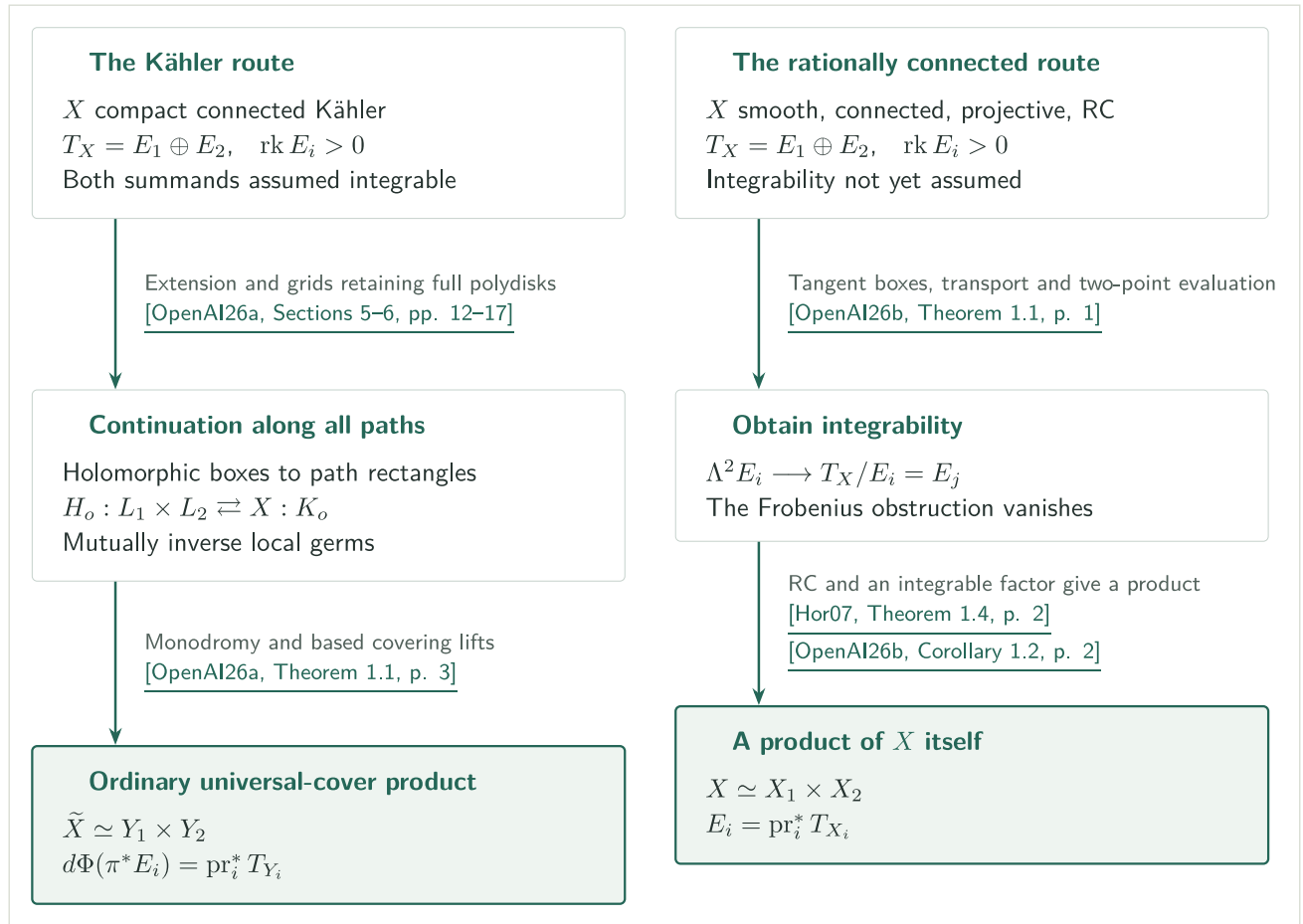
[Read the proof overview](#) →

References on the arrows

- **OpenAI26a** — OpenAI, Universal-cover splitting for compact Kähler manifolds, September 23, 2026, 33 pages. Global holomorphic-box continuation and the ordinary universal-cover product, assuming integrability.
- **OpenAI26b** — OpenAI, Integrability of split tangent bundles on rationally connected manifolds, September 23, 2026, 16 pages. Integrability of the specified splitting from rational connectedness.
- **Hor07** — Andreas Höring, Uniruled varieties with split tangent bundle, arXiv:math/0505327v3 (2017-11-09; internal PDF date November 2, 2018). Theorem 1.4 turns an integrable summand into a product of X itself in the rationally connected projective setting. The external proof itself is outside the checking scope.

[\[OpenAI26a, Theorem 1.1, p. 3\]](#)[\[OpenAI26b, Theorem 1.1, p. 1\]](#)[\[Hor07, Theorem 1.4, p. 2\]](#)

2. Where integrability enters and which space becomes a product



Two routes: integrability is assumed in the Kähler setting and proved in the rationally connected setting

[Open figure ↗](#) [TeX source ↓](#)

The left route promotes local products from two integrable distributions to a global product of the universal cover through all-path continuation. The right route uses tangent boxes of rational curves and two-point evaluation to kill the Lie-bracket obstruction, then applies Höring's product theorem. A tangent box may have critical points; it is not a local product chart before integrability is obtained.

No solid proof-dependency arrow connects the two papers. The left manuscript cites the right main theorem as context in its introduction, not as an input to its main proof. The right Corollary 1.2 uses Höring rather than the left main theorem. This checks the principal connections to the main results; it does not certify the absence of dependencies throughout unexamined references and appendices.

[\[OpenAI26a, Introduction, p. 3\]](#)

[\[OpenAI26b, Corollary 1.2, p. 2\]](#)

The [universal-cover proof overview](#) follows holomorphic extension, area bounds and the two path grids. The [integrability article](#) follows rational reconstruction and holomorphic removal of a formal box, bundle transport and two-point evaluation. Non-uniruled and nef-and-big applications belong to the former; general rationally connected fibres and the rank-two structure belong to the latter. The latter's Corollary 5.1 does not assert integrability or a product for the whole ambient uniruled Kähler manifold.

[OpenAI26b, Corollary 5.1, pp. 12–13]

3. Sources and checking scope

The official field is **Algebraic and complex geometry** (second in official order, entries 032–069). On p.7 of [overview.pdf](#), entry 052 is **Tangent-bundle splittings and universal covers**; [CONTENTS.md](#) uses **Tangent splittings and product decompositions**. The overview title-page date is October 6, 2026; the reference commit is 1581d4cbdd69a0faf34b2089614fe2a814316a6a. Both manuscripts are dated September 23, 2026.

[OpenAI Research Catalog · Catalogue 052 · p. 7](#)

The articles check principal statements, proof overviews for every conclusion, and the statements and uses of direct inputs. The Kollár–Mori theorem statement and page in the universal-cover article, the alternative proof in its Appendices A–D, and complete proofs of external inputs remain unchecked. Each article ends with a reading guide and its precise checking scope. Consistency checks of explanations, diagrams and citations are separate from certifying complete proofs or expert peer review.

Universal-cover splitting for compact Kähler manifolds

Extending holomorphic boxes to obtain a universal-cover product compatible with the specified splitting

The main proof extends holomorphic maps tangent to two integrable tangent summands, placing both leaf paths and arbitrary paths inside rectangles. It constructs continuation of a local product map and its inverse along all paths. Local product coordinates alone do not supply the global conclusion. For a nef and big canonical bundle, approximate Kähler–Einstein metrics supply integrability.

Manuscript author: OpenAI · September 23, 2026 · 33 pages, including references

[Read the manuscript on GitHub ↗](#)

1. Main results

Theorem 1.1 — A universal cover compatible with the specified splitting

Let X be a compact connected Kähler manifold of complex dimension $n \geq 2$, with a specified decomposition

$$T_X = E_1 \oplus E_2$$

into **integrable holomorphic subbundles** of positive ranks r_1, r_2 . For the ordinary universal cover $\pi : \tilde{X} \rightarrow X$, there are connected simply connected complex manifolds Y_1, Y_2 and a biholomorphism $\Phi : \tilde{X} \rightarrow Y_1 \times Y_2$ such that, identifying $T_{\tilde{X}} = \pi^*T_X$ by $d\pi$,

$$d\Phi(\pi^*E_i) = \text{pr}_i^*T_{Y_i}, \quad \dim_{\mathbb{C}} Y_i = r_i \quad (i = 1, 2).$$

The factors are constructed as universal covers of the intrinsic leaves through a common basepoint and may be noncompact. No product decomposition of a finite cover is asserted.

[Theorem 1.1 · p. 3](#)

Corollary 1.2 — Non-uniruled projective manifolds

Let X be a smooth connected non-uniruled projective complex manifold, with a specified holomorphic splitting $T_X = E_1 \oplus E_2$ into positive-rank subbundles. Both summands are integrable, and the ordinary universal cover has a compatible product decomposition as in Theorem 1.1.

Corollary 1.2 · p. 3

Corollary 1.3 — Nef and big canonical bundle

Let X be a smooth connected projective complex manifold of dimension $n \geq 2$ with nef and big K_X . Every specified holomorphic splitting $T_X = E_1 \oplus E_2$ into positive-rank subbundles is integrable and induces the compatible product decomposition of the ordinary universal cover in Theorem 1.1.

Corollary 1.3 · p. 4

References on the arrows

Unprefixed theorem, lemma, section and equation numbers refer to [Universal-cover splitting for compact Kähler manifolds](#). External inputs use the following keys.

- [Iva S. Ivashkovich](#) — Extra extension properties of equidimensional holomorphic mappings: results and open questions: Meromorphic extension to the envelope of holomorphy; removal of indeterminacy is done in the present manuscript. [arXiv:0810.4588v2](#) (2008-11-11); PDF title-page date November 7, 2018.
- [Hor07](#) [Andreas Höring](#) — Uniruled varieties with split tangent bundle: Automatic integrability of a specified split tangent bundle on a non-uniruled projective manifold. [arXiv:math/0505327v3](#) (2017-11-09); PDF title-page date November 2, 2018.
- [Song](#) [Jian Song](#) — Riemannian geometry of Kähler–Einstein currents: Approximate Kähler–Einstein metrics on a crepant resolution and smooth convergence on the regular locus. [arXiv:1404.0445v1](#) (2014).
- [KM98](#) [János Kollár](#); [Shigefumi Mori](#) — Birational Geometry of Algebraic Varieties: Base-point-freeness input; use in the manuscript checked, book statement and page not independently checked. Cambridge Tracts in Mathematics 134 (1998); cited edition, theorem page not independently checked.

Each citation gives the result and pages. When printed pagination differs from the PDF position, both are shown. GitHub previews do not automatically jump to the cited page.

2. Proof of Theorem 1.1 — Global boxes and continuation in both directions

Proof target : Theorem 1.1 · p. 3 See the hypotheses and full conclusion.

Proof route. Integrability first supplies local product charts. Hartogs extension and a mixed-boundary area estimate combine two prescribed axes on their full polydisks. Retaining the entire axis domains permits a finite-grid construction, which continues both the local product map and its inverse along all paths. Monodromy and covering lifts finish the argument. The diagram shows the whole route to the main theorem; its stages are explained immediately below.

Write $P_i : T_X \rightarrow E_i$ for projection along the specified direct sum, not necessarily orthogonal projection. A holomorphic box $f : U \subset \mathbb{C}^a \times \mathbb{C}^b \rightarrow X$ has derivatives in the two parameter blocks tangent to E_1, E_2 , respectively. It need not have full rank or be an embedding.

1 Local products and boxes

$$T_X = E_1 \oplus E_2, \quad E_1, E_2 \text{ integrable}$$

$$df(T\mathbb{C}^a) \subset E_1, \quad df(T\mathbb{C}^b) \subset E_2$$

Tangencies exclude curve fibres of the meromorphic graph
[\[Iva, Theorem 2.2, p. 3\]](#) [Proposition 3.1 · pp. 6–8](#)

2 Remove indeterminacy and cross a mixed boundary

$$H \longrightarrow X \rightsquigarrow Q \longrightarrow X \text{ (holomorphic box)}$$

$$\delta_j/\lambda_j \rightarrow 0, \quad \lambda_j/\sqrt{\delta_j} \rightarrow 0$$

A closed Kähler form bounds area; shells and axes continue the box
[Lemma 4.1; Proposition 4.2; \(4.2\)–\(4.5\) · pp. 8–11](#)
[Propositions 5.1, 5.3; Lemma 5.2 · pp. 12–13](#)

3 Keep prescribed axes on their full polydisks

$$\alpha : S \rightarrow X, \beta : T \rightarrow X \rightsquigarrow F : S \times T \rightarrow X$$

Opposite edges retain holomorphic representatives on the same domains

Reuse entire axes at each tile to handle arbitrary continuous paths
[Proposition 6.1 · pp. 14–15](#)

4 Path grids with prescribed axes or diagonal

$$R(s, 0) = \alpha(s), \quad R(0, t) = \beta(t) \quad \text{or} \quad R(s, s) = \gamma(s)$$

Continue inverse germs along all paths; use monodromy and covering lifts
[Lemma 6.2; Proposition 6.3 · pp. 15–17](#) [Proof of Theorem 1.1 · p. 17](#)

5 Compatible product of the ordinary universal cover

$$\tilde{X} \simeq \tilde{L}_1 \times \tilde{L}_2 = Y_1 \times Y_2$$

$$d\Phi(\pi^* E_i) = \text{pr}_i^* T_{Y_i}, \quad \dim Y_i = r_i$$

Local products, Hartogs extension, mixed-boundary extension, full-polydisk axes, path rectangles and the universal-cover product

[Open figure ↗](#) [TeX source ↓](#)

1. Integrability gives local products; tangencies remove meromorphic indeterminacy

Combining the two Frobenius submersions gives local coordinates in which each E_i is a coordinate block. Transitions locally separate as $(z_1, z_2) \mapsto (h_1(z_1), h_2(z_2))$. This does not assert a product metric.

[Lemma 2.1; Definition 2.2 · pp. 5–6](#)

For a box on a Hartogs figure $H \subset \mathbb{C}^2$, Ivashkovich's theorem gives a **meromorphic** extension to its envelope of holomorphy, a bidisk Q . The input hypotheses are that H is a domain in a Stein manifold and the target is compact Kähler. The manuscript separately proves holomorphicity. If the projection of the graph $\Gamma \subset Q \times X$ had a curve fibre, some component h of a target product chart would vary along it. If h belongs to the first block, a general point is isolated in a level of (s, h) . Near ordinary graph points, however, h is independent of t , so the same type of level contains the t -direction. A level avoiding the boundary of a small ball would then be a positive-dimensional compact analytic set, a contradiction. Properness, density and connectedness of the ordinary graph make the remaining finite fibres singletons; continuous locally bounded coordinate functions remove the indeterminacy. The affine coordinates defining the Hartogs figure need not be the box coordinates (s, t) .

[Iva, Theorem 2.2, p. 3]

Proposition 3.1 · pp. 6–8

2. A closed Kähler form and two scales cross a mixed boundary

At a smooth convex boundary point, assume that the complex tangent has nonzero components in both parameter directions. With v complex tangential and u transverse, the box conditions and the compactness bound $\sup_X \|P_i\| < \infty$ imply $\|df(\partial_{\text{Re } u})\| \leq C\|df(\partial_{\text{Re } v})\|$. On disks $u = x < 0$, $|v| \leq k\sqrt{-x}$, closedness $d\omega = 0$ and Stokes' theorem bound the areas $A(x)$ using the contracting-boundary term.

In slices $x_j = -\delta_j \rightarrow 0$, many disjoint annuli yield one with area tending to zero. The derivative rescaling in Lemma 4.1 places its image in one product chart. Its radius λ_j satisfies both

$$\delta_j/\lambda_j \rightarrow 0, \quad \lambda_j/\sqrt{\delta_j} \rightarrow 0.$$

The first makes the Hartogs shell wider than the remaining distance to the boundary; the second keeps it inside the original tangential disk. Separated target coordinates thicken the shell transversely, and Proposition 3.1 fills it. The Kähler metric is arbitrary: neither orthogonality nor parallelism of the summands is assumed.

Lemma 4.1; Proposition 4.2; (4.2)–(4.5) · pp. 8–11

3. Combine prescribed axes on their entire polydisks

Starting with two holomorphic disks tangent to E_1, E_2 through the same point, extend their box germ over $|s|^{2m} + |t|^{2m} < R^{2m}$. The preceding step treats boundary points off the axes; the prescribed disks treat points on the axes. Extension cannot stop at $R < 1$, and letting $m \rightarrow \infty$

gives the full bidisk. For higher-dimensional axes, first combine radial disks and then reconstruct a finite grid in fixed product charts with parameters. This reconstruction proves holomorphic parameter dependence; pointwise existence of a box would not suffice.

Propositions 5.1, 5.3; Lemma 5.2 · pp. 12–13

4. Two finite grids continue the map and its inverse

Subdivide two leaf paths and express each piece as a holomorphic plaque map evaluated on continuous parameters. After Proposition 5.3 fills a tile, each opposite edge still has a holomorphic representative on the **entire original polydisk**. The prescribed-axis rectangle can therefore be completed without keeping successive images small. For an arbitrary path γ , prescribe diagonal tiles first, then fill bands away from the diagonal to obtain $R(s, s) = \gamma(s)$. This second construction is needed to continue the inverse map along all paths.

Proposition 6.1 · pp. 14–15

5. Universal covers of intrinsic leaves give the biholomorphism and compatibility

Give the leaves L_i through the basepoint their intrinsic plaque topologies, rather than the subspace topology from X . Lemma 6.2 makes them connected complex manifolds with universal covers $Y_i = \tilde{L}_i$. Proposition 6.3 simultaneously continues the inverse local germs $H_o : L_1 \times L_2 \rightarrow X$ and $K_o : X \rightarrow L_1 \times L_2$ along the rectangles.

Lemma 6.2; Proposition 6.3 · pp. 15–17

Monodromy defines maps on the simply connected covers. Lift them with matching basepoints to $\tilde{H} : Y_1 \times Y_2 \rightarrow \tilde{X}$ and $\tilde{K} : \tilde{X} \rightarrow Y_1 \times Y_2$. Both composites are the identity near their basepoints, hence everywhere by connectedness and the identity theorem. The wrong-factor components of $d\tilde{K}$ likewise vanish globally as holomorphic bundle maps. Invertibility of the differential and $\dim Y_i = r_i$ turn the inclusions into equalities. This proves compatibility with the specified embeddings $E_i \subset T_X$, as well as the abstract product isomorphism.

Proof of Theorem 1.1 · p. 17

3. Proof of Corollary 1.2 — Supply integrability from an external theorem

Proof target : Corollary 1.2 · p. 3 See the statement.

1 A non-uniruled projective manifold

X smooth, connected, projective; $T_X = E_1 \oplus E_2$
 Specified holomorphic positive-rank subbundles

Non-uniruledness supplies closure under the Lie bracket
[\[Hor07, Theorem 1.3, p. 1\]](#)

2 Two integrable distributions

$[E_i, E_i] \subset E_i \quad (i = 1, 2)$

Projectivity supplies Kählerness; apply all-path continuation
[Theorem 1.1 · p. 3](#) [Corollary 1.2 · p. 3](#)

3 Compatible universal-cover product

$\tilde{X} \simeq Y_1 \times Y_2, \quad d\Phi(\pi^* E_i) = \text{pr}_i^* T_{Y_i}$

Höring supplies integrability from non-uniruledness; the main theorem then splits the universal cover

[Open figure ↗](#) [TeX source ↓](#)

1. From non-uniruledness to integrability

Höring's Theorem 1.3 makes both factors of a holomorphically split tangent bundle integrable on a non-uniruled projective manifold. The present hypotheses satisfy its assumptions. Its output here is closure under the Lie bracket.

[\[Hor07, Theorem 1.3, p. 1\]](#)

2. From integrability to an ordinary universal-cover product

A smooth projective complex manifold is compact Kähler, so the previous output satisfies Theorem 1.1. The chain is a specified holomorphic splitting, then two integrable distributions, then a compatible universal-cover product. The first step alone is not used to infer the last conclusion in general.

[Corollary 1.2 · p. 3](#)

[Theorem 1.1 · p. 3](#)

4. Proof of Corollary 1.3 — Remove metric error and return to integrability

Proof target : Corollary 1.3 · p. 4 See the statement.

We follow the route in §7. Let $f : X \rightarrow Z$ be the canonical contraction and put $X^\circ = f^{-1}(Z_{\text{reg}})$. The metric parallelism used here holds on X° ; it does not construct a smooth product metric on all of X .

1 The regular canonical locus

K_X nef and big, $f : X \rightarrow Z$, $K_X = f^*K_Z$
 $X^\circ = f^{-1}(Z_{\text{reg}})$

Use approximation and local smooth convergence on a crepant resolution
[\[Song, Equation \(4.28\); Lemma 4.1, pp. 27–28\]](#) [Lemma 7.2 · p. 18](#)

2 Control the error of approximate metrics

$\text{Ric}(\omega_\varepsilon) = -\omega_\varepsilon + \varepsilon \omega_A$, $g_\varepsilon \rightarrow g$ on X°

Add the first Chern degrees to cancel the curvature terms
[Proposition 7.3; \(7.1\) · pp. 18–19](#)

3 Vanishing second fundamental forms

$0 \leq \int_X \sum_j |B_{j,\varepsilon}|^2 dV_\varepsilon \leq \frac{\varepsilon}{(n-1)!} \int_X \omega_A \wedge \omega_\varepsilon^{n-1} = O(\varepsilon)$
 $\nabla_g(E_i|_{X^\circ}) \subset E_i|_{X^\circ}$

Use torsion-freeness and the identity theorem for the holomorphic obstruction
[Proposition 7.3; \(7.1\) · pp. 18–19](#)

4 From a dense open set to global integrability

$\Lambda^2 E_i \rightarrow T_X/E_i$, $u \wedge v \mapsto [u, v] \bmod E_i = 0$

Feed compact Kählerness and integrability into the main theorem
[Theorem 1.1 · p. 3](#) [Proof of Corollary 1.3 · p. 19](#)

5 Pass to the ordinary universal-cover product

$\tilde{X} \simeq Y_1 \times Y_2$, $d\Phi(\pi^* E_i) = \text{pr}_i^* T_{Y_i}$

Canonical model, approximate Kähler–Einstein metrics, integrated second fundamental forms, integrability and the universal-cover product

[Open figure ↗](#) [TeX source ↓](#)

1. The canonical model and approximate Kähler–Einstein metrics

Proposition 7.1 constructs a contraction with $K_X = f^*K_Z$, ample K_Z , canonical Z , and f an isomorphism over Z_{reg} . Semiample uses the cited Kollár–Mori base-point-free theorem: for $D = K_X$ and an integer $a > 1$, the divisor $aD - K_X = (a - 1)K_X$ is nef and big. Its use in the manuscript was checked; the book's theorem statement and exact page have not been independently checked here.

Proposition 7.1 · pp. 17–18

Song's approximation applies to this smooth crepant resolution. For $0 < \varepsilon < 1$ it supplies metrics satisfying

$$\text{Ric}(\omega_\varepsilon) = -\omega_\varepsilon + \varepsilon\omega_A, \quad [\omega_\varepsilon] = 2\pi c_1(K_X) + \varepsilon[\omega_A].$$

The next step uses smooth convergence on compact subsets of X° to a negative Kähler–Einstein metric g . Song's additional conclusion identifying its metric completion with Z is not needed for this main-text route.

[Song, Equation (4.28); Lemma 4.1, pp. 27–28]

Lemma 7.2 · p. 18

2. Vanishing second fundamental forms imply global integrability

Let $P_{j,\varepsilon}$ be g_ε -orthogonal projection onto E_j and $B_{j,\varepsilon}$ its second fundamental form. These projections differ from those along the specified direct sum in the main proof. Integrating and adding the two subbundle-curvature identities cancels degrees through

$c_1(E_1) + c_1(E_2) = c_1(T_X)$. Since $C_\varepsilon = P_{1,\varepsilon} + P_{2,\varepsilon} - I$ satisfies $\text{tr } C_\varepsilon = 0$ and $-I \leq C_\varepsilon \leq I$,

$$0 \leq \int_X \sum_{j=1}^2 |B_{j,\varepsilon}|^2 dV_\varepsilon \leq \frac{\varepsilon}{(n-1)!} \int_X \omega_A \wedge \omega_\varepsilon^{n-1} = O(\varepsilon).$$

The bound comes from a polynomial in cohomology classes. Smooth convergence on X° forces both limiting second fundamental forms to vanish, so the connection of g preserves both factors. Its torsion-free property implies integrability on X° . The holomorphic Frobenius obstruction

$$\Lambda^2 E_j \longrightarrow T_X/E_j, \quad u \wedge v \longmapsto [u, v] \bmod E_j$$

vanishes on a dense open set, hence on all of X . This is where the metric argument returns to the holomorphic hypothesis of the main theorem.

Proposition 7.3; (7.1) · pp. 18–19

3. Theorem 1.1 supplies the global product

Integrability and projectivity now permit Theorem 1.1, giving the ordinary universal-cover product compatible with the specified splitting. No completeness of the metric on X° is assumed here, and de Rham's decomposition is not directly applied to that parallelism. Appendices A–D give a different global continuation mechanism through collapsed canonical fibres; it is separate from the route explained here.

Proof of Corollary 1.3 · p. 19

5. Which papers supply which steps

Input

[Iva, Theorem 2.2, p. 3]

Used at

Proposition 3.1

Content and hypotheses

Meromorphic extension from a Hartogs domain in a Stein manifold to a compact Kähler target

Checking scope

External statement and application checked; holomorphic removal belongs to this manuscript

Input

[Hor07, Theorem 1.3, p. 1]

Used at

Corollary 1.2

Content and hypotheses

Integrability of the specified splitting under non-uniruledness and projectivity

Checking scope

External statement and application on p.3 checked

Input

KM98: Theorem 3.3

Used at

Proposition 7.1

Content and hypotheses

Semiamplessness of K_X

Checking scope

Use in manuscript checked; book statement and page not independently checked

Input

[Song, Equation (4.28); Lemma 4.1, pp. 27–28]

Used at

Lemma 7.2 $\rightarrow$ Proposition 7.3

Content and hypotheses

Approximate metrics on a crepant resolution and smooth convergence off the exceptional locus

Checking scope

Input statement and applicability checked; external proofs outside scope

The companion article [on integrability on rationally connected manifolds](#) treats another setting supplying the main theorem's integrability hypothesis. Its Theorem 1.1 is cited in this manuscript's introduction, but it is not a direct input to this manuscript's main proof or the two corollaries above.

[Introduction · p. 3](#)

6. Return to the sources

- [Theorem 1.1 · p. 3](#) — Integrability as a hypothesis and compatibility with the specified splitting
- [Proposition 3.1 · pp. 6–8](#) — Removal of exceptional fibres of the meromorphic graph
- [Lemma 4.1; Proposition 4.2; \(4.2\)–\(4.5\) · pp. 8–11](#) — Closed Kähler form, area estimate and the two scales

- Propositions 5.1, 5.3; Lemma 5.2 · pp. 12–13 — Extension retaining the entire polydisk axes
- Proposition 6.1 · pp. 14–15 — The prescribed-axis and prescribed-diagonal grids
- Proof of Theorem 1.1 · p. 17 — From two-way continuation to the product isomorphism and compatibility
- Proposition 7.3; (7.1) · pp. 18–19 — From metric parallelism to global integrability

We read the principal statements and the proof text in §§2–7, checking all conclusions of the main theorem, the connections to both corollaries, and the statements and uses of the Ivashkovich, Horing and Song inputs. The statement and exact page of Kollr–Mori’s base-point-free theorem remain unchecked. The alternative proof in Appendices A–D, proofs of external theorems, background references and all underlying analytic-set and covering-space theory have not been independently verified. This article organizes the manuscript’s arguments; it does not certify the complete proofs or constitute expert peer review.

Integrability of split tangent bundles on rationally connected manifolds

Separating formal tangent directions, extending a rational map, and killing the bracket tensors

The manuscript proves integrability of both summands of a specified holomorphic tangent-bundle splitting on a smooth rationally connected projective manifold. Its key construction extends a rational curve on the diagonal to a map whose two factor directions lie in the two summands. Relative connections on this map move the bracket tensor into a trivial target bundle; a two-point family of rational curves then forces it to vanish. A global product requires the additional input of Höring’s product theorem.

Manuscript author: OpenAI · September 23, 2026 · 16 pages, including references

[Read the manuscript on GitHub ↗](#)

1. Main results

Theorem 1.1 — Automatic integrability of both specified summands

Let X be a smooth connected projective complex manifold of dimension at least **2**.

Suppose that two general points of X lie on the image of a morphism $\mathbb{P}^1 \rightarrow X$. Here “two general points” means every pair in some nonempty Zariski open subset of $X \times X$. For every specified holomorphic decomposition

$$T_X = E_1 \oplus E_2$$

into subbundles of positive rank, both E_1 and E_2 are integrable. The embeddings $E_i \subset T_X$ are part of the specified splitting.

[Theorem 1.1 · p. 1](#)

Corollary 1.2 — A product of the manifold compatible with its specified splitting

Under the hypotheses of Theorem 1.1, there are smooth projective manifolds X_1, X_2 and an isomorphism $X \simeq X_1 \times X_2$ under which

$$E_i = \text{pr}_i^* T_{X_i} \quad (i = 1, 2)$$

as subbundles. This is a product decomposition of X itself.

Corollary 1.2 · p. 2

Corollary 5.1 — Two fibrations from the rationally connected quotient

Let X be a connected uniruled compact Kähler manifold with a holomorphic splitting $T_X = V_1 \oplus V_2$. Let Z be a general fiber of its rationally connected quotient and put $W_i = T_Z \cap V_i|_Z$. For each $i = 1, 2$, there is an almost-holomorphic fibration $\varphi_i: X \dashrightarrow Y_i$ whose general fiber F_i is rationally connected and, for compatible choices $F_i \subset Z$, satisfies

$$T_{F_i} = W_i|_{F_i} \subset V_i|_{F_i}.$$

If $W_i = 0$, one may take $\varphi_i = \text{id}_X$ with point fibers.

If in addition $\text{rank } V_1 = 2$, exactly one of the following holds.

1. $W_1 = V_1|_Z$. There is an analytic fiber bundle $X \rightarrow Y$ with rationally connected general fiber and $T_{X/Y} = V_1$.
2. W_1 is a line bundle. There is an equidimensional holomorphic map $X \rightarrow Y$ whose general fiber F is a rational curve with $T_F \subset V_1|_F$.
3. $W_1 = 0$, equivalently $T_Z \subset V_2|_Z$.

Corollary 5.1 · pp. 12–13

References on the arrows

Unprefixed theorem, lemma, section and equation numbers refer to Integrability of split tangent bundles on rationally connected manifolds. External inputs use the following keys.

- [Hor07](#) Andreas H\"oring — Uniruled varieties with split tangent bundle: Compatible product from an integrable specified splitting on a rationally connected manifold. arXiv:math/0505327v3 (PDF margin: 9 Nov 2017; internal date: November 2, 2018); numbering of this preprint, not the journal version.

- [Hor08](#) Andreas Hring — The structure of uniruled manifolds with split tangent bundle: General RC quotient fibers, their projectivity and tangent splitting, almost-holomorphic fibrations and the rank-two trichotomy. Osaka Journal of Mathematics 45 (2008), 1067–1084; published text, repository PDF with one cover sheet.

Each citation gives the result and pages. When printed pagination differs from the PDF position, both are shown. GitHub previews do not automatically jump to the cited page.

2. Theorem 1.1 — Proof overview of the main theorem

Proof target : [Theorem 1.1](#) · p. 1

Proof route. Write P_i for the projectors of the splitting in [Theorem 1.1](#). Without using rational connectedness, extend any $g: C = \mathbb{P}^1 \rightarrow X$ to a map $F: S = C \times C \rightarrow X$ with separated tangent directions. The construction is explained in the [section on Proposition 2.1](#). Relative connections identify F^*E_i with a pullback from the i th factor. Rational connectedness enters only afterwards, through submersive two-point evaluation.

1 A splitting and a rational curve

$$T_X = E_1 \oplus E_2, \quad g : C = \mathbb{P}^1 \rightarrow X$$

Fix the projectors P_i ; integrability is not assumed.

Rationalize the formal solution and contract exceptional curves.

[Proposition 2.1 · p. 3](#)

2 Separate the tangent directions

$$F : C \times C \rightarrow X, \quad F|_{\Delta} = g, \quad dF(T^{(i)}) \subset F^*E_i$$

Pull back the projected-bracket connection and transport on $\mathbb{P}^1$ fibers.

[Proposition 3.1; equation \(3.2\) · p. 10](#)

3 Separate input and target on a curve

$$L_i = \{s_j = b\} : \quad F^*E_i|_{L_i} \simeq g^*E_i, \quad F^*E_j|_{L_i} \simeq \mathcal{O}_C \otimes E_{j,g(b)}$$

Use rational connectedness here to choose a submersive two-point family.

[Lemma 4.1 · p. 11](#)

4 Test the bracket by two-point evaluation

$$u(a) = v(a) = 0, \quad \text{span}\{u(b)\} = E_{i,g(b)}$$

$$\sigma_{u,v} = \Theta_j^{-1}(F^*B_i)(\Theta_i u \wedge \Theta_i v)$$

A constant section of a trivial bundle: $\sigma_{u,v}(a) = 0 \Rightarrow \sigma_{u,v}(b) = 0$.

Transport is the identity on the diagonal; extend vanishing by the identity principle.

[Theorem 1.1 \(proof\) · p. 11](#)

5 Both specified summands are integrable

$$B_i(g(b)) = 0 \quad (i = 1, 2) \implies B_1 = B_2 = 0 \text{ on } X$$

A map with separated tangent directions and relative transport lead to the vanishing of both bracket tensors

[Open figure ↗](#) [TeX source ↓](#)

1. Trivialize the complementary bundle on a factor curve

Take F satisfying $F|_{\Delta} = g$ and $dF(T^{(i)}) \subset F^*E_i$, where $T^{(i)} = \text{pr}_i^* T_C$. The map may have critical points: a tangent box does not supply coordinates or a local product decomposition of X .

For $i \neq j$, $\xi \in E_j$ and $e \in E_i$, the formula $\nabla_{\xi}^i e = P_i[\xi, e]$ is tensorial in the direction and satisfies the Leibniz rule in e . This partial connection requires no integrability of either bundle. The tangency of F pulls it back to a holomorphic connection on F^*E_i relative to pr_i . The construction does not invert dF and remains defined at critical points.

On each $\mathbb{P}^1$ fiber its curvature is a holomorphic two-form and vanishes. Simple connectedness makes parallel transport path-independent. Transport starting on the diagonal varies holomorphically and gives the isomorphisms of Proposition 3.1:

$$\Theta_i: \text{pr}_i^*(g^*E_i) \xrightarrow{\sim} F^*E_i, \quad \Theta_i|_{\Delta} = \text{id}.$$

Flatness here concerns the restriction to the curve fibers; no flat connection on all of X has been produced.

[Proposition 2.1 · p. 3](#) · [Proposition 3.1; equation \(3.2\) · p. 10](#)

2. Produce sections vanishing at one point and spanning at the other

Fix distinct $a, b \in C$. Lemma 4.1 supplies a nonempty family $G: C \times T \rightarrow X$ for which $(\text{ev}_a, \text{ev}_b): T \rightarrow X \times X$ is a submersion. Appendix B constructs countably many families using polynomial presentations in projective space. Their two-point images contain general pairs. If all images had deficient rank, their union would have measure zero, so some family has full rank.

For $g = G(\cdot, \tau)$, differentiating in a parameter direction gives a global section of g^*T_X . Surjectivity of two-point evaluation, followed by projection with P_i , shows that global sections of g^*E_i vanishing at a have values spanning $E_{i,g(b)}$. The tangent boxes of the separate curves need not themselves be chosen as a holomorphic family.

[Lemma 4.1 · p. 11](#) · [Appendix B; Lemma 4.1 \(proof\) · pp. 14–15](#)

3. Kill both brackets by reducing their values to constant sections

The obstruction to integrability is the holomorphic bundle morphism

$$B_i: \bigwedge^2 E_i \longrightarrow E_j, \quad B_i(e, e') = P_j[e, e'] \quad (j \neq i).$$

On $L_i = \{s_j = b\} \subset S$, the map Θ_i identifies the input bundle with g^*E_i , while Θ_j identifies the target with $\mathcal{O}_C \otimes E_{j,g(b)}$. For global sections u, v of g^*E_i vanishing at a ,

$$\sigma_{u,v} = \Theta_j^{-1}((F^*B_i)(\Theta_i u \wedge \Theta_i v)) \in H^0(C, \mathcal{O}_C \otimes E_{j,g(b)})$$

is constant. It vanishes at a and therefore at b . At (b, b) , one has $F(b, b) = g(b)$ and the transport maps are the identity, so $B_i(g(b))(u(b) \wedge v(b)) = 0$. The spanning property gives $B_i(g(b)) = 0$. The image of ev_b is nonempty and open, and the identity principle on connected X makes B_i vanish everywhere. Applying the argument for both $i = 1, 2$ proves integrability of both specified subbundles.

[Theorem 1.1 \(proof\) · p. 11](#)

3. Proposition 2.1 — From formal tangency to a global tangent box

Proof target : Proposition 2.1 · p. 3

Proposition 2.1 — Tangent box

Let X be a smooth projective complex manifold with a fixed holomorphic splitting $T_X = E_1 \oplus E_2$. Put $C = \mathbb{P}^1$, $S = C \times C$, let $\Delta \subset S$ be the diagonal, and set $T^{(i)} = \text{pr}_i^* T_C$. Every holomorphic map $g: C \rightarrow X$ admits a holomorphic map $F: S \rightarrow X$ satisfying

$$F|_{\Delta} = g, \quad dF(T^{(i)}) \subset F^* E_i \quad (i = 1, 2).$$

Neither summand is assumed integrable.

Proposition 2.1 · p. 3

1 A formal Cauchy problem

$$u = s, v = t - s : \quad \widehat{F}(u, 0) = g(u), \quad \partial_v \widehat{F} = P_2(\widehat{F}) \partial_u \widehat{F}$$

Idempotence makes the formal equation equivalent to both tangencies.

[Lemma 2.2; equation \(2.2\) · pp. 4–5](#)

2 Glue the unique formal solution

$$\widehat{F} = \sum_{k \geq 0} f_k(u) v^k, \quad f_1 = P_2(g)g'$$

Recursion determines coefficients; uniqueness handles changes of charts.

The filtration with $I/I^2 = \mathcal{O}_{\mathbb{P}^1}(-2)$ bounds formal sections.

[Lemma 2.3; equations \(2.3\)–\(2.4\) · pp. 6–7](#)

3 Reconstruct rational coordinate ratios

$$h^0(\Delta, M_m) \leq m^2 + O(m+1) < 2(m+1)^2$$

$$AS_0 + BS_\nu = 0 \quad \Rightarrow \quad S_\nu/S_0 = -A/B$$

Cancel common normal powers; decreasing base-ideal colength resolves the map.

[Lemma 2.3; equations \(2.3\)–\(2.4\) · pp. 6–7](#)

[Lemma A.1 · pp. 13–14](#)

4 Cancel locally, then resolve by point blowups

$$R : S \dashrightarrow X, \quad \mu : S' \rightarrow S, \quad F' : S' \rightarrow X$$

$$(P_1 dF') \wedge d(s \circ \mu) = 0, \quad (P_2 dF') \wedge d(t \circ \mu) = 0$$

Cancel exceptional vanishing orders; the map is constant on connected fibers.

[Lemma 2.5; Proposition 2.1 \(proof\) · pp. 8–9](#)

5 Contract exceptional curves and descend

$$P_1 dF'|_{T_H} = P_2 dF'|_{T_H} = 0 \quad \Rightarrow \quad dF'|_{T_H} = 0$$

$$F : S \rightarrow X, \quad F|_\Delta = g, \quad dF(T^{(i)}) \subset F^* E_i$$

A section count rationalizes the formal solution, and contraction of exceptional curves gives a global holomorphic map

[Open figure ↗](#) [TeX source ↓](#)

1. Solve a formal Cauchy problem using the projectors

With $u = s, v = t - s$ near the diagonal, the required tangencies are equivalent to

$$\widehat{F}(u, 0) = g(u), \quad \partial_v \widehat{F} = P_2(\widehat{F}) \partial_u \widehat{F}.$$

In $\widehat{F} = \sum f_k(u) v^k$, comparison of coefficients determines $(k+1)f_{k+1}$ from previous coefficients and their u derivatives. Uniqueness glues the local solutions across coordinate changes. No convergence in the normal direction is asserted at this stage.

Lemma 2.2; equation (2.2) · pp. 4–5

2. Turn positivity of the diagonal's normal bundle into a section bound

For a projective embedding $X \subset \mathbb{P}^N$, let L be the formal pullback of the hyperplane bundle, with $L|_\Delta \simeq \mathcal{O}_C(d)$. Since $I/I^2 \simeq \mathcal{O}_C(-2)$, the filtration by normal vanishing order of $M_m = L \otimes \widehat{\mathcal{O}}_S(m, m)$ yields

$$\dim H^0(\Delta, M_m) \leq \sum_{k \geq 0} \max\{d + 2m - 2k + 1, 0\} = m^2 + O(m + 1).$$

Pairs of ordinary polynomial sections form a space of dimension $2(m + 1)^2$. This forces a nonzero relation $AS_0 + BS_\nu = 0$ between formal homogeneous coordinates, expressing $S_\nu/S_0 = -A/B$ as a rational function. Choose a point of the diagonal where the leading denominator coefficients are nonzero. Canceling common powers of the normal parameter then gives a holomorphic map near that point with the same formal expansion. The manuscript does not invoke an external G3 theorem for this step.

Lemma 2.3; equations (2.3)–(2.4) · pp. 6–7 · Remark 2.4 · p. 8

3. The two tangencies force zero derivative on exceptional curves

Resolve the resulting rational map by finitely many point blowups $\mu: S' \rightarrow S$, obtaining $F': S' \rightarrow X$. Appendix A proves finite termination through a decreasing local base-ideal colength. The tangencies give global identities

$$(P_1 dF') \wedge d(s \circ \mu) = 0, \quad (P_2 dF') \wedge d(t \circ \mu) = 0.$$

At a general point of an exceptional curve H , write $s \circ \mu - s(p) = r^\alpha h$ and $t \circ \mu - t(p) = r^\beta \tilde{h}$. Cancel the common powers and restrict to $r = 0$. Both $P_1 dF'|_{T_H}$ and $P_2 dF'|_{T_H}$ vanish, so $dF'|_{T_H} = 0$. Every exceptional component maps to a point. Connectedness of the fibers of μ allows F' to descend to S , and removability at isolated points makes the descended map holomorphic. The identity principle extends its diagonal value and tangencies globally.

Lemma A.1 · pp. 13–14 · Lemma 2.5; Proposition 2.1 (proof) · pp. 8–9

4. Corollary 1.2 — From integrability to a compatible product

Proof target : Corollary 1.2 · p. 2

The target is Corollary 1.2. The preceding argument establishes integrability of specified subbundles; obtaining a global product is a separate step.

1 Rational connectedness and the specified splitting

X smooth, connected, projective, $T_X = E_1 \oplus E_2$
 Theorem 1.1 makes both E_i integrable.

Supply at least one integrable summand to Höring's product theorem.
[\[Hor07, Theorem 1.4, p. 2\]](#)

2 A global product compatible with the subbundles

$X \simeq X_1 \times X_2$, $E_i = \text{pr}_i^* T_{X_i}$

Restrict a projective embedding of X to the closed slices.
[Corollary 1.2 · p. 2](#)

3 The factors are smooth and projective

$X_1 \simeq X_1 \times \{x_2\} \subset X$, $X_2 \simeq \{x_1\} \times X_2 \subset X$

Apply Höring's product theorem after integrability and realize the factors as projective slices

[Open figure ↗](#) [TeX source ↓](#)

1. Apply the product theorem and realize the factors as projective slices

Höring's Theorem 1.4 starts with a specified tangent splitting on a rationally connected projective manifold and requires at least one integrable summand. It produces $X \simeq X_1 \times X_2$ compatible with both summands. The manuscript's Theorem 1.1 supplies the integrability hypothesis. The factors are smooth and occur as closed complex submanifolds of X by taking slices of the product. Restriction of a projective embedding of X proves that each factor is projective.

Compatibility here concerns the specified holomorphic tangent subbundles; it does not assert an orthogonal product for a metric. The product of X itself should also be distinguished from one obtained only after passing to a finite or universal cover.

[\[Hor07, Theorem 1.4, p. 2\]](#) · [Corollary 1.2 · p. 2](#)

5. Corollary 5.1 — Apply the theorem on a general RC fiber and separate the ranks

Proof target : [Corollary 5.1 · pp. 12–13](#)

The target is [Corollary 5.1](#). The ambient X is not assumed projective or rationally connected. Theorem 1.1 is applied to a general fiber Z of its RC quotient.

1 Choose a general RC quotient fiber

X uniruled compact Kähler, $T_X = V_1 \oplus V_2$

Z smooth, connected, Kähler, RC

Use the RC quotient, recalled Campana projectivity, and projectors preserving T_Z .

[Hor08, Definition 2.9; Theorem 2.10, print p. 1070 / PDF p. 5]

[Hor08, remark after Definition 2.8, print p. 1070 / PDF p. 5]

[Hor08, Proposition 3.16, print p. 1073 / PDF p. 8]

2 Split the tangent bundle of a projective fiber

Z projective, $T_Z = W_1 \oplus W_2$, $W_i = T_Z \cap V_i|_Z$

Apply the main theorem precisely when its dimension and positive-rank hypotheses hold.

Theorem 1.1 · p. 1

3 Integrability on the general fiber

W_1, W_2 both integrable

Use the main theorem for positive ranks; otherwise the summands are 0, T_Z .

Meet the integrability premise of the structure theorems; use identity for zero rank.

[Hor08, Theorems 1.3–1.4, print p. 1068 / PDF p. 3]

[Hor08, Theorem 1.3 (proof), print p. 1074 / PDF p. 9]

4 Two fibrations and the rank-two trichotomy

$\varphi_i : X \dashrightarrow Y_i$, $T_{F_i} = W_i|_{F_i}$; $W_i = 0 \Rightarrow \varphi_i = \text{id}$

$\text{rk } V_1 = 2 : \text{rk } W_1 = 2, 1, 0$

Analytic fiber bundle / rational-curve general fiber / $T_Z \subset V_2|_Z$.

Projectivity and splitting on the general RC fiber supply the integrability hypothesis of the structure theorems

Open figure ↗ TeX source ↓

1. Establish the hypotheses on the general fiber

Using Definition 2.9 and Theorem 2.10 in Höring’s published version, choose Z over a regular value in the fibration locus. It is a smooth connected compact submanifold, inherits a Kähler form, and is rationally connected. Campana’s result recalled immediately after Definition 2.8 makes Z projective.

Proposition 3.16 says that the ambient projectors P_i preserve T_Z . Their restrictions are complementary holomorphic idempotents of constant rank on connected Z . Their images are therefore subbundles W_i and $T_Z = W_1 \oplus W_2$. If both ranks are positive, then $\dim Z \geq 2$ and Theorem 1.1 applies. Otherwise the two summands are 0, T_Z and integrability already holds.

[Hor08, Definition 2.9; Theorem 2.10, print p. 1070 / PDF p. 5] ·

[Hor08, remark after Definition 2.8, print p. 1070 / PDF p. 5] ·

[Hor08, Proposition 3.16, print p. 1073 / PDF p. 8] · Theorem 1.1 · p. 1

2. Pass from integrability on general fibers to two fibrations

The integrability hypothesis of Höring’s Theorem 1.3 now holds. For both $i = 1, 2$, it gives an almost-holomorphic fibration with $T_{F_i} = W_i|_{F_i}$. The published convention permits the identity map, and its proof handles $W_i = 0$ by id_X . This case is treated separately from Theorem 1.1, which requires positive ranks. Integrability of W_i does not imply integrability of the ambient V_i in general.

[Hor08, Theorems 1.3–1.4, print p. 1068 / PDF p. 3] · [Hor08, Theorem 1.3 (proof), print p. 1074 / PDF p. 9]

3. Obtain all three alternatives under the rank-two hypothesis

If $\text{rank } V_1 = 2$, the rank of W_1 is 2, 1, or 0. Under the same integrability premise, Höring’s Theorem 1.4 gives, respectively, an analytic fiber bundle with $T_{X/Y} = V_1$, an equidimensional holomorphic map with rational-curve general fiber, or $T_Z \subset V_2|_Z$. The ranks are mutually exclusive and exhaustive, proving the “exactly one” assertion in Corollary 5.1.

[Hor08, Theorems 1.3–1.4, print p. 1068 / PDF p. 3] · Corollary 5.1 · pp. 12–13

6. Which papers supply which steps

Input

[Hor07, Theorem 1.4, p. 2]

Used at

Corollary 1.2

Role and hypotheses

A compatible product for a specified splitting on projective RC X with at least one integrable summand

Checking scope

Statement in the specified version and manuscript application checked

Input

[Hor08, Definition 2.9; Theorem 2.10, print p. 1070 / PDF p. 5] · [Hor08, remark after Definition 2.8, print p. 1070 / PDF p. 5]

Used at

Corollary 5.1 inputs

Role and hypotheses

Choose the general RC fiber as a smooth compact Kähler, hence projective, manifold

Checking scope

Published conventions and statements, including the recalled Campana result, checked

Input

[Hor08, Proposition 3.16, print p. 1073 / PDF p. 8]

Used at

Splitting on the fiber

Role and hypotheses

Ambient projectors preserve T_Z and give complementary subbundles

Checking scope

Statement and preservation argument checked

Input

[Hor08, Theorems 1.3–1.4, print p. 1068 / PDF p. 3] · [Hor08, Theorem 1.3 (proof), print p. 1074 / PDF p. 9]

Used at

Fibrations and trichotomy

Role and hypotheses

Integrable W_i give the fibrations; zero rank allows identity, and $\text{rank } V_1 = 2$ gives three cases

Checking scope

Statements, applicability and identity case checked

The tangent-box construction for the main theorem is internal to the manuscript. Formal-function and G3 theory are cited as background, but Lemma 2.3 does not claim to use a G3 theorem as a direct input. Checking external statements and their applications is distinct from verifying their proofs.

Remark 2.4 · p. 8

7. Return to the sources

- Proposition 2.1 · p. 3 — The tangent-box statement and its three-stage construction.
- Lemma 2.3; equations (2.3)–(2.4) · pp. 6–7 — The section count that reconstructs rational coordinate functions.
- Proposition 3.1; equation (3.2) · p. 10 — Relative connections without an integrability hypothesis and diagonal normalization.
- Theorem 1.1 (proof) · p. 11 — Constant sections and two-point evaluation kill both bracket tensors.
- Corollary 1.2 · p. 2 — The external product input and projectivity of the factors.
- Corollary 5.1 · pp. 12–13 — Reapply the main theorem to general RC quotient fibers.

The statements, the proof text in §§2–5, and Appendices A–B were read to trace formal-to-rational reconstruction, contraction of exceptional curves, relative transport, and two-point evaluation leading to bracket vanishing. The input statements in the specified versions of the two Horing papers were compared with their uses. Campana’s projectivity result was checked as recalled in Horing’s published paper; its original proof was not read.

This does not constitute a complete independent reconstruction of all technical details of formal gluing, decreasing base ideals, or holomorphic parameter dependence of transport. External proofs and their further dependencies were not recursively verified. It is neither expert peer review nor certification of the entire argument. Cross-catalog connections beyond those recorded remain uninvestigated.