

1 (1, 3) x 4 行列

(2) 0

(3) $\begin{pmatrix} 3 & -1 & 2 & -5 \end{pmatrix}$

(4) $\begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$

2 (1) $\begin{pmatrix} 5 & -2 \\ 0 & 1 \end{pmatrix} - 3 \begin{pmatrix} 3 & 4 \\ 1 & -2 \end{pmatrix}$

$$= \begin{pmatrix} 5 & -2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -9 & -12 \\ -3 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -14 \\ -3 & 7 \end{pmatrix}$$

(2) $\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 2 & 4 \\ -1 & -5 \end{pmatrix} + \begin{pmatrix} 5 & -1 \\ 6 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 3 \\ 5 & -7 \end{pmatrix}$$

$$(3) \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}^3 + 3 \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 125 & 0 \\ 0 & 8 \end{pmatrix} + \begin{pmatrix} 15 & 3 \\ 3 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 140 & 3 \\ 3 & 14 \end{pmatrix}$$

$$\boxed{3} \quad (1) \det \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} = 3 \times 4 - 2 \times 1 = 10$$

$$(2) \begin{pmatrix} 2 & 1 \\ -5 & 3 \end{pmatrix}^{-1} = \frac{1}{2 \times 3 - (-5) \times 1} \begin{pmatrix} 3 & -1 \\ 5 & 2 \end{pmatrix}$$

$$= \frac{1}{11} \begin{pmatrix} 3 & -1 \\ 5 & 2 \end{pmatrix}$$

$$(3) \begin{pmatrix} 100 & 99 \\ 99 & 100 \end{pmatrix}^{-1} = \frac{1}{100^2 - 99^2} \begin{pmatrix} 100 & -99 \\ -99 & 100 \end{pmatrix}$$

$$= \frac{1}{199} \begin{pmatrix} 100 & -99 \\ -99 & 100 \end{pmatrix}$$

④ 特征値と特征ベクトル

$$\textcircled{1} \det(A - tE_2) = \det \begin{pmatrix} 4-t & 2 \\ 0 & 3-t \end{pmatrix} = 0$$

$t \in \mathbb{C}$

$$(4-t)(3-t) - 2 \times 0 = 0$$

$$\Rightarrow (4-t)(3-t) = 0$$

$$\Rightarrow t = 3, 4$$

$$\textcircled{2} Av = 3v \text{ に対する } v = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \text{ は } 1 \text{ 次元空間}$$

$$\begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 3x_1 \\ 3y_1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \leftarrow \begin{pmatrix} 4 \\ -2 \end{pmatrix} \text{ などでも可} \\ \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ は除外} \right)$$

$$Av = 4v \text{ に対する } v = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \text{ は } 1 \text{ 次元空間}$$

$$\begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4x_2 \\ 3y_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\textcircled{3} P = \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \text{ etc etc}$$

$$P^{-1}AP = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \text{ etc etc}$$

$$\text{etc etc } (P^{-1}AP)^n = \begin{pmatrix} 3^n & 0 \\ 0 & 4^n \end{pmatrix} \text{ etc etc}$$

$$(P^{-1}AP)^n = P^{-1}A^nP$$

$$A^n = P \begin{pmatrix} 3^n & 0 \\ 0 & 4^n \end{pmatrix} P^{-1}$$

$$P^{-1} = \frac{1}{2 \times 0 - (-1) \times 1} \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} \text{ etc}$$

$$\begin{aligned} A^n &= P \begin{pmatrix} 3^n & 0 \\ 0 & 4^n \end{pmatrix} P^{-1} \\ &= \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3^n & 0 \\ 0 & 4^n \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} \end{aligned}$$

$$= \begin{pmatrix} 2 \cdot 3^n & 4^n \\ -3^n & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4^n & 2(4^n - 3^n) \\ 0 & 3^n \end{pmatrix} //$$

$$(2) \text{ ① } \det(B - t E_2) = \det \begin{pmatrix} 13-t & -30 \\ 5 & -12-t \end{pmatrix} \text{ E.C.}$$

$$\Rightarrow (13-t)(-12-t) - (-30) \cdot 5 = 0$$

$$\Rightarrow -156 - t + t^2 + 150 = 0$$

$$\Rightarrow t^2 - t - 6 = 0$$

$$\Rightarrow (t-3)(t+2) = 0$$

$$t = 3, -2$$

$$(2) \quad Bx = 3x \quad \&\&\< \quad (x = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix})$$

$$\begin{pmatrix} 13 & -30 \\ 5 & -12 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = 3 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \quad \&\&\<$$

$$\begin{pmatrix} 10 & -30 \\ 5 & -15 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \therefore \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$Bx = -2x \quad \&\&\<$$

$$\begin{pmatrix} 13 & -30 \\ 5 & -12 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = -2 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \quad \&\&\<$$

$$\begin{pmatrix} 15 & -30 \\ 5 & -10 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \therefore \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$(3) \quad P = \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \quad \&\&\<$$

$$P^T A P = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix} \quad \&\&\<$$

$$\&\&\< \quad (P^T A P)^n = \begin{pmatrix} 3^n & 0 \\ 0 & (-2)^n \end{pmatrix}$$

$$P^{-1} = \frac{1}{3-2} \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix} \quad \&\&\<$$

$$A^n = P \cdot (P^T A P)^n \cdot P^{-1}$$

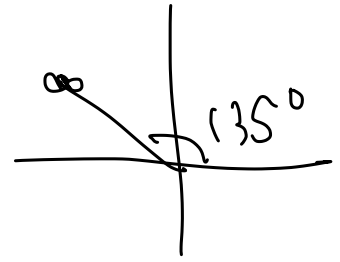
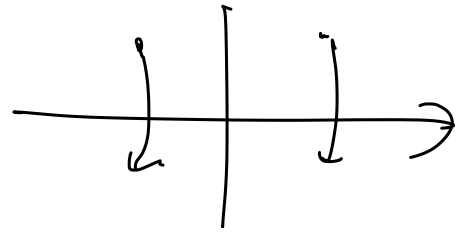
$$= \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 3^n & 0 \\ 0 & (-2)^n \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3^{n+1} & 2 \cdot (-2)^n \\ 3^n & (-2)^n \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3^{n+1} + (-2)^{n+1} & -2 \cdot 3^{n+1} + 6(-2)^n \\ 3^n - (-2)^n & -2 \cdot 3^n + 3 \cdot (-2)^n \end{pmatrix}$$

5. (1) 2軸に對しての
鏡映(反転)

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



135度反時計回し回転

$$B = \begin{pmatrix} \cos 135^\circ & -\sin 135^\circ \\ \sin 135^\circ & \cos 135^\circ \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$$

2. 2軸鏡映 A

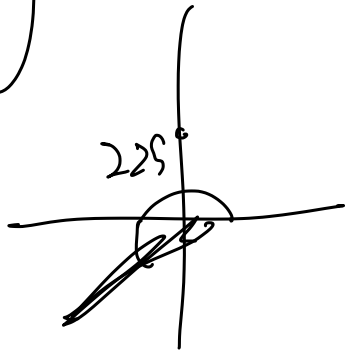
135度反時計回し回転 B

$$BA = \begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

//

(2) 光軸鏡映 h_2 135度反時計回りに
光軸鏡映 $y/2$

$$\begin{aligned}
 ABA &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \\
 &= \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}
 \end{aligned}$$


$$= \begin{pmatrix} \cos 225^\circ & -\sin 225^\circ \\ \sin 225^\circ & \cos 225^\circ \end{pmatrix}$$

$$\boxed{\frac{15}{2}} \quad \alpha = 225$$

$$\boxed{\text{証明}} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos \alpha^\circ & -\sin \alpha^\circ \\ \sin \alpha^\circ & \cos \alpha^\circ \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ = \begin{pmatrix} \cos 360^\circ - \alpha^\circ & -\sin(360^\circ - \alpha^\circ) \\ \sin(360^\circ - \alpha^\circ) & \cos(360^\circ - \alpha^\circ) \end{pmatrix}$$

$$\textcircled{6-1} \begin{pmatrix} 1 & 0 & 3 \\ 4 & -1 & 3 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & -2 \\ 17 & -5 & 4 \\ 1 & 0 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 & -2 & -6 \\ -11 & 4 & -8 & -4 & -6 \\ 14 & -10 & 8 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 & -8 \\ -4 & 9 & -18 \\ 15 & -10 & 6 \end{pmatrix}$$

$$\textcircled{2} \begin{pmatrix} 2 & -1 \\ 4 & 0 \\ 3 & -5 \end{pmatrix} \left\{ 2 \begin{pmatrix} 0 & 1 & -2 \\ 17 & -5 & 4 \end{pmatrix} - 3 \begin{pmatrix} 1 & -2 & 6 \\ 4 & -1 & 5 \end{pmatrix} \right\}$$

$$= \begin{pmatrix} 2 & -1 \\ 4 & 0 \\ 3 & -5 \end{pmatrix} \left\{ \begin{pmatrix} 0 & 2 & -4 \\ 14 & -10 & 8 \end{pmatrix} + \begin{pmatrix} -3 & 6 & -18 \\ -12 & 3 & -15 \end{pmatrix} \right\}$$

$$= \begin{pmatrix} 2 & -1 \\ 4 & 0 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} -3 & 8 & -22 \\ 2 & -7 & -7 \end{pmatrix}$$

$$= \begin{pmatrix} -6-2 & +16+17 & -44+17 \\ -12 & 32 & -88 \\ -9-10 & 24+35 & -66+35 \end{pmatrix}$$

$$= \begin{pmatrix} -8 & 23 & -31 \\ -12 & 32 & -88 \\ -19 & 59 & -31 \end{pmatrix}$$

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⑦: $\bullet A 1 \times 3, A 1 \times 3$ より AA 定義できない

$\bullet A 1 \times 3, B 3 \times 3$ より $AB 1 \times 3$ 行列

$$\bullet AB = (-1 \ 2 \ -5) \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{pmatrix} = (-2 \ 6 \ -27)$$

$\bullet A 1 \times 3, C 2 \times 3$ より AC 定義できない

$\bullet A 1 \times 3, D 3 \times 1$ より $AD 1 \times 1$ 行列

$$(-1 \ 2 \ -5) \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} = (4 + 6 - 5) = (5)$$

$\bullet B 3 \times 3, A 1 \times 3$ より BA 定義できない

$\bullet B 3 \times 3, B 3 \times 3$ より $B \cdot B 3 \times 3$ 行列

$$BB = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 9 & 0 & 12 \\ 0 & 9 & 0 \\ 24 & 0 & 33 \end{pmatrix}$$

$\bullet B 3 \times 3, C 2 \times 3$ より BC 定義できない

$\bullet B 3 \times 3, D 3 \times 1$ より $BD 3 \times 1$ 行列

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 9 \\ -11 \end{pmatrix}$$

* $C 2 \times 3$ $A 1 \times 3$ より CA 定義できない

* $C 2 \times 3$ $B 3 \times 3$ より CB 行列

$$\begin{pmatrix} -2 & 5 & 3 \\ 1 & -3 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 10 & 15 & 11 \\ 1 & -9 & 2 \end{pmatrix}$$

* $C 2 \times 3$, $C 2 \times 3$ より CC 定義できない

* $C 2 \times 3$ $D 3 \times 1$ より CD 行列

$$\begin{pmatrix} -2 & 5 & 3 \\ 1 & -3 & 0 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 8+15+3 \\ 4-9 \end{pmatrix} = \begin{pmatrix} 26 \\ -5 \end{pmatrix}$$

* $D 3 \times 1$ $A 1 \times 3$ より DA 行列

$$\begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} (-2-5) = \begin{pmatrix} 4-8-20 \\ 3-6-15 \\ 1-2-5 \end{pmatrix}$$

* $D 3 \times 1$ $B 3 \times 3$ より DB 定義できない

* $D 3 \times 1$ $C 2 \times 3$ より DC 定義できない

* $D 3 \times 1$ $D 3 \times 1$ より DD 定義できない

$$\text{of } Z = \frac{1}{2} \text{ of } \frac{1}{2}$$

$$\bullet AB = (-2 \mid 6 \mid -27) \bullet AD = (5)$$

$$\bullet BB = \begin{pmatrix} 9 & 0 & 12 \\ 0 & 9 & 0 \\ 29 & 0 & 33 \end{pmatrix}$$

$$\bullet BD = \begin{pmatrix} 7 & 2 \\ 9 \\ -11 \end{pmatrix}$$

$$\bullet CB = \begin{pmatrix} 10 & 15 & 11 \\ 1 & -9 & 2 \end{pmatrix}$$

$$CD = \begin{pmatrix} 26 \\ 43 \end{pmatrix}$$

$$\bullet DA = \begin{pmatrix} 4 & -8 & 20 \\ -3 & 6 & -15 \\ 1 & 2 & -5 \end{pmatrix}$$

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