1-1

(A, E_3) を簡約化して $(E_3\ B)$ になったとき

$B = A^{-1}$ であることを使うと。

$$\begin{pmatrix} 2 & -1 & 0 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} & 0 & 1 \end{pmatrix}$$

$-2\ 1\ 0\ -1\ 0\ 0$

$-1\ \frac{1}{2}\ 0\ -\frac{1}{2}\ 0\ 0$

$$\rightarrow \begin{pmatrix} 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} & 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{pmatrix}$$

$1\ 1\ -1$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & -1 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{pmatrix}$$

$\sqrt{2}$

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 1 & -1 & 0 \end{pmatrix}$$

1-2

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$-2\ -4\ -6\ -2\ 0\ 0$

$-3\ -6\ -9\ -3\ 0\ 0$

$$\rightarrow \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -4 & -4 & -2 & 1 & 0 \\ 0 & -4 & -8 & -3 & 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & \frac{2}{4} & -\frac{1}{4} & 0 \\ 0 & 1 & 2 & \frac{3}{4} & 0 & -\frac{1}{4} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 1 & 0 & \frac{2}{4} & 0 \\ 0 & 1 & 1 & \frac{2}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{pmatrix}$$

$0 \ -2 \ -2 \ -1 \ +\frac{2}{4} \ 0$

$0 \ -1 \ -1 \ -\frac{2}{4} \ \frac{1}{4} \ 0$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{4} & -\frac{2}{4} & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{pmatrix}$$

$-1 \ \frac{1}{4} \ -\frac{1}{4} \ \frac{1}{4}$

$$\frac{1}{4} \begin{pmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

2-1

$$\begin{vmatrix} 5 & -3 & 14 \\ -5 & 6 & 7 \\ 10 & 3 & -7 \end{vmatrix} = 5\times 3\times 7 \begin{vmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= 105\times \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & 3 \\ 0 & 3 & -5 \end{vmatrix}$$

2 −2 4

$$= 105 \times \begin{vmatrix} 1 & 3 \\ 3 & -5 \end{vmatrix}$$

$$\begin{array}{r} 105 \\ \times\ 14 \\ \hline 420 \\ 105 \end{array}$$

$$= 105\times(-(9+5))$$

$$= 105\times(-14) = -1470$$ //

2-2

$$\begin{vmatrix} 2 & 3 & 1 & 0 \\ 3 & 0 & 1 & 0 \\ 2 & 0 & 4 & -1 \\ 1 & 0 & 1 & 2 \end{vmatrix} = - \begin{vmatrix} 3 & 2 & 1 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 2 & 4 & -1 \\ 0 & 1 & 1 & 2 \end{vmatrix}$$

$$= -3\begin{vmatrix} 3 & 1 & 0 \\ 2 & 4 & -1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= 3\begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & -1 \\ 3 & 1 & 0 \end{vmatrix}$$

2 2 4
3 3 6

$$= 3 \times \begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & -5 \\ 0 & -2 & -6 \end{vmatrix}$$

$$= 3 \times \begin{vmatrix} 2 & -5 \\ -2 & -6 \end{vmatrix}$$

$$= 3 \times (-12 - 10) = 3 \times (-22)$$

$$= -66 \text{ //}$$

2-3

$$\begin{vmatrix} 3 & 1 & 3 & 5 \\ 6 & 2 & 2 & 6 \\ -3 & 1 & 0 & 1 \\ 3 & 1 & 1 & 6 \end{vmatrix} = 2 \begin{vmatrix} 3 & 1 & 3 & 5 \\ 3 & 1 & 1 & 3 \\ -3 & 1 & 0 & 1 \\ 3 & 1 & 1 & 6 \end{vmatrix}$$

$$= 6 \begin{vmatrix} 1 & 1 & 3 & 5 \\ 1 & 1 & 1 & 3 \\ -1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 6 \end{vmatrix}$$

$$= 6 \begin{vmatrix} 1 & 1 & 3 & 5 \\ 0 & 0 & -2 & -2 \\ 0 & 2 & 3 & 6 \\ 0 & 0 & -2 & 1 \end{vmatrix}$$

$$= 6 \begin{vmatrix} 0 & -2 & -2 \\ 2 & 3 & 6 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= -6 \begin{vmatrix} 2 & 3 & 6 \\ 0 & -2 & -2 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= -12 \begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix}$$

$$= -12(-2-4) = 72$$

$\boxed{2-4)}$

$$\begin{vmatrix} 3 & 5 & 7 & 0 & -1 \\ 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 7 & 2 & 2 \\ 0 & 0 & 3 & 0 & 5 \\ 0 & 0 & 5 & 1 & -1 \end{vmatrix} = -\begin{vmatrix} 5 & 3 & 7 & 0 & -1 \\ 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 7 & 2 & 2 \\ 0 & 0 & 3 & 0 & 5 \\ 0 & 0 & 5 & 1 & -1 \end{vmatrix}$$

$$= -5\begin{vmatrix} 2 & 0 & 1 & 0 \\ 1 & 7 & 2 & 2 \\ 0 & 3 & 0 & 5 \\ 0 & 5 & 1 & -1 \end{vmatrix} + \begin{vmatrix} 3 & 7 & 0 & -1 \\ 1 & 7 & 2 & 2 \\ 0 & 3 & 0 & 5 \\ 0 & 5 & 1 & -1 \end{vmatrix}$$

$$= -10\begin{vmatrix} 7 & 2 & 2 \\ 3 & 0 & 5 \\ 5 & 1 & -1 \end{vmatrix} + 5\begin{vmatrix} 0 & 1 & 0 \\ 3 & 0 & 5 \\ 5 & 1 & -1 \end{vmatrix}$$

$$+ 3\begin{vmatrix} 7 & 2 & 2 \\ 3 & 0 & 5 \\ 5 & 1 & -1 \end{vmatrix} - \begin{vmatrix} 7 & 0 & -1 \\ 3 & 0 & 5 \\ 5 & 1 & -1 \end{vmatrix}$$

$$= -7\begin{vmatrix} 7 & 2 & 2 \\ 3 & 0 & 5 \\ 5 & 1 & -1 \end{vmatrix} + 5\begin{vmatrix} 0 & 1 & 0 \\ 3 & 0 & 5 \\ 5 & 1 & -1 \end{vmatrix} - \begin{vmatrix} 7 & 0 & -1 \\ 3 & 0 & 5 \\ 5 & 1 & -1 \end{vmatrix}$$

$$= -7(50 + 6 - 35 + 6)$$

$\begin{array}{r} 15 \\ 12 \\ \hline \end{array}$

$$+5(25+3)$$

$$-1(-3-35)$$

$$= -7 \times 27 + 5 \times 28 + 38$$

$$= -1\overset{4}{8}9 + 140 + 38$$

$$= -189 + 178 = -11 \;//$$

3-1

$$\begin{vmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 2 & -1 & x & 3 \\ x & -2 & 2 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 & 2 \\ -1 & 0 & -1 & -1 \\ 1 & 2 & x & 3 \\ 2 & x & 2 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & x-1 & 1 \\ 0 & x-2 & 0 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 1 \\ 1 & x-1 & 1 \\ x-2 & 0 & 0 \end{vmatrix}$$

$$= - \begin{vmatrix} x-2 & 0 & 0 \\ 1 & x-1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= -(x-2) \begin{vmatrix} x-1 & 1 \\ 0 & 1 \end{vmatrix} = -(x-2)(x-1)_{//}$$

3-2 Aが逆行列を持たない

$\Longleftrightarrow$ Aが正則でない

$\Longleftrightarrow$ $\det A = 0$ より

$-(x-2)(x-1) = 0$ なるx、つまり

$x = 2, 1$ が答えである //

$\boxed{4-1}$ $\det(AB) = (\det A)(\det B)$ より

$\det(ABC)$

$= (\det A)(\det B)(\det C)$

$= (\det A)(\det C)(\det B)$

$= \det(ACB)$ //

$\boxed{4-2}$ A が正則 $\Longleftrightarrow$ $\det A \neq 0$ より

AB が正則 $\Rightarrow$ $\det(AB) \neq 0$

$\Rightarrow$ $(\det A)(\det B) \neq 0$

$\Rightarrow$ $\det A \neq 0$ かつ $\det B \neq 0$

$\Rightarrow$ A が正則 かつ B が正則 //