

$$\textcircled{1} \begin{pmatrix} 1 & 2 \\ -4 & -1 \\ 5 & -2 \end{pmatrix} + 2 \begin{pmatrix} 2 & -1 \\ 0 & 4 \\ -7 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ -4 & -1 \\ 5 & -2 \end{pmatrix} + \begin{pmatrix} 4 & -2 \\ 0 & 8 \\ -14 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ -4 & 7 \\ -9 & -2 \end{pmatrix}$$

$$\textcircled{2} 3 \times \begin{pmatrix} 2 & -1 & 4 \\ 0 & 3 & -5 \end{pmatrix} - 2 \left\{ \begin{pmatrix} 0 & 1 & -2 \\ 7 & -5 & 4 \end{pmatrix} - 3 \begin{pmatrix} 1 & -2 & 6 \\ 4 & -1 & 5 \end{pmatrix} \right\}$$

$$= \begin{pmatrix} 6 & -3 & 12 \\ 0 & 9 & -15 \end{pmatrix} - 2 \times \left\{ \begin{pmatrix} 0 & 1 & -2 \\ 7 & -5 & 4 \end{pmatrix} + \begin{pmatrix} -3 & 6 & -18 \\ -12 & 3 & -15 \end{pmatrix} \right\}$$

$$= \begin{pmatrix} 6 & -3 & 12 \\ 0 & 9 & -15 \end{pmatrix} - 2 \times \begin{pmatrix} -3 & 7 & -20 \\ -5 & -2 & -11 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -3 & 12 \\ 0 & 9 & -15 \end{pmatrix} + \begin{pmatrix} 6 & -14 & 40 \\ 10 & 4 & 22 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & -17 & 52 \\ 10 & 13 & 7 \end{pmatrix}$$

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② A 1×3 , B 3×3 かつ AB 1×3 行列

$$AB = (-1 \ 2 \ -5) \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{pmatrix} = (-2 \ 6 \ -27)$$

B 3×3 , A 1×3 かつ AB 定義できない

A 1×3 , C 2×3 かつ AC 定義できない

C 2×3 , A 1×3 かつ CA 定義できない

A 1×3 , D 3×1 かつ AD 1×1 行列

$$(-1 \ 2 \ -5) \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} = (4 + 6 - 5) = (5)$$

D 3×1 , A 1×3 かつ DA 3×3 行列

$$\begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} (-1 \ 2 \ -5) = \begin{pmatrix} 4 & -8 & 20 \\ -3 & 6 & -15 \\ -1 & 2 & -5 \end{pmatrix}$$

B 3×3 , C 2×3 かつ BC 定義できない

C 2×3 , B 3×3 かつ CB 2×3 行列

$$\begin{pmatrix} -2 & 5 & 3 \\ 1 & -3 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 10 & 15 & 11 \\ 1 & -9 & 2 \end{pmatrix}$$

• B 3×3 D 3×1 $\Rightarrow (BD)$ 3×1 \checkmark 行列

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 9 \\ -11 \end{pmatrix}$$

• D 3×1 B 3×3 $\Rightarrow (DB)$ 3×3 \checkmark 行列

• C 2×3 D 3×1 $\Rightarrow (CD)$ 2×1 \checkmark 行列

$$\begin{pmatrix} -2 & 5 & 3 \\ 1 & -3 & 0 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 8+15+3 \\ 4-9 \end{pmatrix} = \begin{pmatrix} 26 \\ -13 \end{pmatrix}$$

• D 3×1 C 2×3 $\Rightarrow (DC)$ 3×3 \checkmark 行列

$\Rightarrow Z = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

• $AB = \begin{pmatrix} -2 & 6 & -2 \end{pmatrix}$ • $AD = \begin{pmatrix} 5 \end{pmatrix}$

• $DA = \begin{pmatrix} 4 & -8 & 20 \\ -3 & 6 & -15 \\ 1 & 2 & -5 \end{pmatrix}$ • $BD = \begin{pmatrix} -2 \\ 9 \\ -11 \end{pmatrix}$

• $CB = \begin{pmatrix} 10 & 15 & 11 \\ 1 & -9 & 2 \end{pmatrix}$

• $CD = \begin{pmatrix} 26 \\ -13 \end{pmatrix}$

○ 60.

3-11

$$\begin{pmatrix} 1 & 1 & 5 & 0 & 3 \\ 3 & 1 & 9 & 1 & 8 \\ 2 & 0 & 4 & 1 & 5 \\ 2 & 1 & 7 & 1 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 5 & 0 & 3 \\ 0 & -2 & -6 & 1 & -1 \\ 0 & -2 & -6 & 1 & -1 \\ 0 & -1 & -3 & 1 & 1 \end{pmatrix} \quad \begin{array}{l} -3-3-15 \ 0-9 \\ -2-2-10 \ 0-6 \end{array}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & 5 & 0 & 3 \\ 0 & -2 & -6 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -3 & 1 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & 5 & 0 & 3 \\ 0 & -1 & -3 & 1 & 1 \\ 0 & -2 & -6 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 5 & 0 & 3 \\ 0 & 1 & 3 & -1 & -1 \\ 0 & 2 & 6 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{array}{l} +(-3)1 \\ -2-6 \ 22 \end{array}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 2 & 1 & 4 \\ 0 & 1 & 3 & -1 & -1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\rightarrow -7 \ 3$

(3-2)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow \Rightarrow 2$

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4-1 行列の逆行列

① 拡大係数行列は $\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 2 & 2 & 0 \end{pmatrix}$ の逆行列

② 行列を簡約化する

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{array}{l} -2 \quad -4 \quad -2 \quad 0 \\ -1 \quad -2 \quad -1 \quad 0 \end{array}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right) \quad \begin{array}{l} c \\ d \end{array} \quad //$$

③ $Cx = d \quad x \in C, \quad //$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
 $x \in C$

よって $\left. \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{array} \right\}$

$I = \{0\} //$

4-2

① 積大係數行列付

$$\begin{pmatrix} 1 & 1 & 5 & 0 & 3 \\ 3 & 1 & 9 & 1 & 8 \\ 2 & 0 & 4 & 1 & 5 \\ 2 & 1 & 7 & 1 & 9 \end{pmatrix} \quad 2^{\text{nd}} \text{ 行}$$

② 行列を簡約化せよ

$$\begin{pmatrix} 1 & 1 & 5 & 0 & 3 \\ 2 & 1 & 7 & 1 & 9 \\ 3 & 1 & 9 & 1 & 8 \end{pmatrix} \xrightarrow{3-1} \begin{pmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{matrix} c \\ d \end{matrix}$$

③ $Cx = d$ を解け

$$\text{つまり} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix} \text{ を解く}$$

$$\text{つまり} \begin{cases} x_1 + 2x_3 = 1 \\ x_2 + 3x_3 = 2 \\ x_4 = 3 \\ 0 = 0 \end{cases}$$

$$\text{よって} \begin{cases} x_1 = 1 - 2t \\ x_2 = 2 - 3t \\ x_3 = t \\ x_4 = 3 \end{cases} \quad (t \text{ 実数})$$

4-3

① 拡大係数行列 $\left(\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & 1 & \\ 2 & -1 & 2 & 2 & 6 & 2 & \\ 3 & 2 & -4 & -3 & -9 & 3 & \end{array} \right)_{2 \times 1}$

② 二枚を簡約化する

$$\left(\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & 1 & \\ 2 & -1 & 2 & 2 & 6 & 2 & \\ 3 & 2 & -4 & -3 & -9 & 3 & \end{array} \right) \rightarrow \left(\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & 1 & \\ 0 & -3 & 6 & 0 & 0 & 0 & -2-2 \text{ 4-2 6-2} \\ 0 & -1 & 2 & -6 & -18 & 0 & -3-3 \text{ 6-3-9-3} \end{array} \right)$$

$$\rightarrow \left(\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & 1 & \\ 0 & 1 & -2 & 0 & 0 & 0 & \\ 0 & -1 & 2 & -6 & -18 & 0 & \end{array} \right)$$

$$\rightarrow \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 1 & 3 & 1 & \\ 0 & 1 & -2 & 0 & 0 & 0 & \\ 0 & 0 & 0 & -6 & -18 & 0 & \end{array} \right)$$

$$\rightarrow \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 1 & 3 & 1 & \\ 0 & 1 & -2 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 3 & 0 & \end{array} \right)$$

$$\rightarrow \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & \\ 0 & 1 & -2 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 3 & 0 & \end{array} \right)$$

C d

$$(3) \quad Cx = d \quad f \in C$$

$$\text{例 11} \quad \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad f \in C$$

$$\text{例 11} \quad \begin{cases} x_1 = 1 \\ x_2 = 2x_3 \\ x_4 + 3x_5 = 0 \end{cases} \quad \begin{matrix} = 1 \\ = 0 \end{matrix} \quad f \in C$$

$$\boxed{\begin{matrix} x_4 \\ x_5 \end{matrix}} \quad \begin{cases} x_1 = 1 \\ x_2 = 2s \\ x_3 = s \\ x_4 = -3t \\ x_5 = t \end{cases} \quad (s, t) \text{ 実数.}$$

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