1-1

$$\int_0^{\frac{\pi}{2}} (\cos x)^2 dx = \int_0^{\frac{\pi}{2}} \frac{\cos 2x + 1}{2} dx$$

$$= \left[\frac{\sin 2x}{4} + \frac{x}{2} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{4}$$

1-2

$$\int_0^{\pi} (\sin x)^2 (\cos x) dx = \left[\frac{(\sin x)^3}{3} \right]_0^{\pi}$$

$$= 0$$

1-3 $\int_1^2 (\log x)^2 dx = \int_1^2 x' (\log x)^2 dx$

$$= \left[x (\log x)^2 \right]_1^2 - \int_1^2 x \cdot 2 (\log x) \cdot \frac{1}{x} dx$$

$$= 2(\log 2)^2 - 2\int_1^2 \log x \, dx$$

$$= 2(\log 2)^2 - 2\left[x \log x - x \right]_1^2$$

$$= 2(\log 2)^2 - 2\left((2\log 2 - 2) - (1\log 1 - 1)\right)$$

$$= 2(\log 2)^2 - 2(2\log 2 - 1)$$

$$= 2(\log 2)^2 - 4\log 2 + 2 \quad //$$

$\boxed{1-4}$

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx \qquad \frac{dx}{dt} = \frac{1}{(\cos t)^2}$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+(\tan t)^2}\,\frac{dt}{(\cos t)^2}$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} dt = \left[t\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\pi \quad //$$

$\boxed{2-1}$ $f(x,y) = \frac{x}{y} + \log x$

$$\frac{\partial f}{\partial x} = \frac{1}{y} + \frac{1}{x}$$

$$\frac{\partial f}{\partial y} = \frac{-x}{y^2}$$

2-2 $f(x,y) = e^{xy} \sin x$

$$\frac{\partial f}{\partial x} = y e^{xy} \sin x + e^{xy} \cos x.$$

$$\frac{\partial f}{\partial y} = x e^{xy} \sin x$$

B $f(x)=x^2+xy+2y^2-4y$

判別 $\frac{\partial f}{\partial x}=\frac{\partial f}{\partial y}=0$ なる点をもとめる

$$\frac{\partial f}{\partial x}=2x+y=0$$

$$\frac{\partial f}{\partial y}=x+4y-4=0$$ より,

$$x=-\frac{4}{7} \quad y=\frac{8}{7}$$

判別 $D_f=\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right)-\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$

$$=2\cdot 4-1^2$$

$$=7>0$$

$$\frac{\partial^2 f}{\partial x^2}=2>0$$ より

$\left(-\frac{4}{7}, \frac{8}{7}\right)$ で極小値をとる

極小値 $f\left(-\frac{4}{7},\frac{8}{7}\right)$

$$=\left(-\frac{4}{7}\right)^2+\left(-\frac{4}{7}\right)\left(\frac{8}{7}\right)+2\left(\frac{8}{7}\right)^2-4\left(\frac{8}{7}\right)$$

$$= \frac{1}{49}(16-32+2\times8\times8-4\times8\times7)$$

$$= \frac{1}{49}(-16+4\times8\times(4-7))$$

$$= \frac{1}{49}(-16+4\times8\times(-3))$$

$$\begin{array}{r} 12 \\ \times\ 8 \\ \hline 96 \end{array}$$

$$= \frac{-1}{49}(16+96)$$

42

$$= \frac{-112}{49} = -\frac{16}{7}$$

//

$\boxed{答}$ $\left(-\frac{4}{7}, \frac{8}{7}\right)$で極小値 $-\frac{16}{7}$をとる。

④

<u>条件1</u> $\frac{\partial g}{\partial x} = 2x$ $\frac{\partial g}{\partial y} = 2y$ より

$\frac{\partial g}{\partial x} = \frac{\partial g}{\partial y} = g = 0$ なる点は存在しない

<u>条件2</u>

$F = f - tg$ とおく

$= (x^2+xy+y^2) - t(x^2+y^2-1)$

$\frac{\partial F}{\partial x} = 2x+y-2tx = 0$ ①

$\frac{\partial F}{\partial y} = 2y+x-2ty = 0$ ②

$\frac{\partial F}{\partial t} = -(x^2+y^2-1) = 0$ ③

をとく

① + ② より

$3x+3y-2t(x+y)=0$

$(x+y)(3-2t) = 0$

$x+y=0$ ならば ③ より $2x^2=1$ より

$(x, y) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$x+y \neq 0$ ならば $2t=3$ より

① ⇒ $x = y$.

よって $(x, y) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ となる

よって $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ が候補になる

$f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{3}{2}$ 最大

$f\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{1}{2}$

$f\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$ 　最小 より

$f\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = \frac{3}{2}$ 最大

答 $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ で最大値 $\frac{3}{2}$.

$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ で最小値 $\frac{1}{2}$ となる.